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Identification of Linear Model Parameters and Uncertainties for an Aircraft Turbofan Engine

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Introduction

THE identification of turbofan engine dynamics as a multivariable piecewise linear model, along with modeling uncertainties from turbofan engine control system nonlinear model data, is necessary for developing advanced algorithms of sensor/actuator failure detection/isolation/accommodation and robust optimal control. Some gas turbine models have been identified. A number of papers and reports on engine identification and parameter estimation have been discussed in the survey by Merrill et al.[1] An algorithm based on least-squares estimation and nonlinear dynamic filtering was highlighted. The model was multivariable, and noise was introduced to simulate stochastic I/O data. The maximum likelihood method was applied to simulated open-loop and actual closed-loop engine data.[2] A nonlinear programming technique was used to estimate matrix parameters of a state-space aircraft model.[3] A two-step estimation approach for nonlinear systems with unknown process and measurement-noise covariances was applied to simulated aircraft response data.[4] A parameter identification algorithm based on smoothing test data with successively improved sets of system model parameters[5] and the maximum likelihood method using the V-Lambda square-root filtering technique[6] decrease the numerical difficulties. There also were some identification efforts in the frequency domain[7] and even in the quantification of parametric uncertainty.[8] In the time domain, the identification of model parameters and associated uncertainties was made for robust[9] or reconfigurable[10] control design.

In this work, a new estimation procedure is used to estimate both unknown piecewise linear model matrix parameters and matrix parameters limiting modeling uncertainties. These uncertainties are the differences between nonlinear and linear models and not uncertainties in the nonlinear model itself. Therefore, the purpose of this work was the development of an identification method for linear model parameters and uncertainties in the time domain from nonlinear simulation data, its application, and demonstration using a real case.

Identification of a Piecewise Linear Model

The detailed nonlinear models of different aircraft engines and, in particular, of a modern twin-spool afterburning turbofan engine, may be presented approximately as

$$\dot{\boldsymbol{x}}^{\text{abs}} = \boldsymbol{f}(\boldsymbol{x}^{\text{abs}}, \boldsymbol{u}^{\text{abs}}, \text{ALT}, \text{MN}) \tag{1}$$

$$\boldsymbol{y}^{\text{abs}} = \boldsymbol{g}(\boldsymbol{x}^{\text{abs}}, \boldsymbol{u}^{\text{abs}}, \text{ALT}, \text{MN}) \tag{2}$$

where $\boldsymbol{x}^{\text{abs}}$ is a state vector, $\boldsymbol{u}^{\text{abs}}$ is a control vector, and $\boldsymbol{y}^{\text{abs}}$ is an output vector; and system functions $\boldsymbol{f}$ and $\boldsymbol{g}$ are nonlinear real-value vector functions. A model of this type is used to obtain response data at arbitrary operating points. Three variables—altitude (ALT), Mach number (MN), and power-lever angle (PLA)—are sufficient to completely define each operating point.

A set of linear models, along with a description of modeling errors (uncertainties) at several selected operating points, is a piecewise linear model that can describe engine behavior at all operating points. Each linear model is assumed to be described by the following discretized equations:

$$\boldsymbol{x}(k+1) = (A \pm \Delta A)\boldsymbol{x}(k) + B\boldsymbol{u}(k) \tag{3}$$

$$\boldsymbol{y}(k) = (C \pm \Delta C)\boldsymbol{x}(k) + D\boldsymbol{u}(k) \tag{4}$$

where $\boldsymbol{x}$ is an n-dimensional state deviation vector, $\boldsymbol{u}$ is an m-dimensional control deviation vector, and $\boldsymbol{y}$ is an l-dimensional output deviation vector that does not include state deviation vector components. In other words, $\boldsymbol{x} = \boldsymbol{x}^{\text{abs}} - \boldsymbol{x}^{\text{sta}}$, $\boldsymbol{u} = \boldsymbol{u}^{\text{abs}} - \boldsymbol{u}^{\text{sta}}$, and $\boldsymbol{y} = \boldsymbol{y}^{\text{abs}} - \boldsymbol{y}^{\text{sta}}$ are the vectors of deviations from steady-state values; $\boldsymbol{x}^{\text{sta}}$, $\boldsymbol{u}^{\text{sta}}$, and $\boldsymbol{y}^{\text{sta}}$ are the vectors of steady-state values that correspond to one of the operating points; $k = 0, \ldots, N-1$ corresponds to $t_0, \ldots, t_{N-1}$, at $t_{k+1} = t_k + \Delta t$; and Δt is a uniform sampling time.

We want to estimate unknown matrices A, B, C, and D and unknown matrices with positive elements $\Delta A^{\max}$, $\Delta C^{\max}$ based on the assumption that

$$0 \le \Delta a_{ij} \le \Delta a_{ij}^{\max}, \qquad i = 1, \ldots, n, \quad j = 1, \ldots, n \tag{5}$$

$$0 \le \Delta c_{ij} \le \Delta c_{ij}^{\max}, \qquad i = 1, \ldots, l, \quad j = 1, \ldots, n \tag{6}$$

We first determine static model matrix parameters for each selected point of the static curve. If the states are steady near the point,

$$\boldsymbol{x} = K^x \boldsymbol{u} \tag{7}$$

$$\boldsymbol{y} = K^y \boldsymbol{u} \tag{8}$$

and the columns of matrices K^x and K^y may be determined from the detailed nonlinear engine simulation of response to given scalar step inputs by each component of the control vector. Let us assume that

$$B = (I - A)K^x \tag{9}$$

$$D = K^y - CK^x \tag{10}$$

The matrices A, C and $\Delta A^{\max}, \Delta C^{\max}$ then may be estimated from the discrete data of the nonlinear model simulated response $\tilde{\boldsymbol{x}}(k), k = 0, \ldots, N$ and $\tilde{\boldsymbol{y}}(k), k = 0, \ldots, N-1$ to $\tilde{\boldsymbol{u}}(k), k = 0, \ldots, N-1$ corresponding to PLA step inputs near a selected operating point. Obviously, we cannot use the data corresponding to $k = 0$ if $\tilde{\boldsymbol{x}}(k) = 0$. This estimation implies solving a linear programming problem for each row of matrices A, $\Delta A^{\max}$ (A_i, $\Delta A_i^{\max}$, $i = 1, \ldots, n$):

$$A_i, \Delta A_i^{\max} : \min \left\{ \delta, \delta \ge \sum_{j=1}^{n} \Delta a_{ij}^{\max} |\tilde{x}_j(k)|, \quad k = 1, \ldots, N-1 \right.$$
$$\sum_{j=1}^{n} \Delta a_{ij}^{\max} |\tilde{x}_j(k)| \ge \left| \tilde{x}_i(k+1) - A_i \tilde{\boldsymbol{x}}(k) - \left(K_i^x - A_i K^x\right)\tilde{\boldsymbol{u}}(k) \right|, \quad k = 1, \ldots, N-1$$
$$\left. \Delta a_{ij}^{\max} \ge 0, \quad j = 1, \ldots, n \right\} \tag{11}$$

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and C, $\Delta C^{\max}$ (C_i, $\Delta C_i^{\max}$, $i = 1, \ldots, l$)

$$C_i, \Delta C_i^{\max} : \min \left\{ \begin{aligned} &\delta, \delta \geq \sum_{j=1}^{n} \Delta c_{ij}^{\max} |\tilde{x}_j(k)|, \quad k = 1, \ldots, N-1 \\ &\sum_{j=1}^{n} \Delta c_{ij}^{\max} |\tilde{x}_j(k)| \geq \left| \tilde{y}_i(k) - C_i \tilde{\boldsymbol{x}}(k) \right. \\ &\qquad \left. - \left(K_i^y - C_i K^x\right) \tilde{\boldsymbol{u}}(k) \right|, \quad k = 1, \ldots, N-1 \\ &\Delta c_{ij}^{\max} \geq 0, \quad j = 1, \ldots, n \end{aligned} \right\} \tag{12}$$

Thus, we can obtain matrices A, $\Delta A^{\max}$, C, $\Delta C^{\max}$, and then B and D by Eqs. (9) and (10) for all selected operating points. It is clear that solving the linear programming problem in Eqs. (11) and (12) will not lead to better estimates of A, B, C, and D than linearizations of the nonlinear model at each operating point. Only the presented approach, however, allows us to also estimate $\Delta A^{\max}$ and $\Delta C^{\max}$ using any additional linear constraints for some elements of all of these matrices.

Simulation Results

This section describes the application of the developed twin-spool, afterburning turbofan engine control system identification method at altitude ALT = 0 and Mach number MN = 0 corresponding to sea-level takeoff conditions. Two state variables were considered: engine fan speed *NF* and compressor speed *NC*. Four control variables were considered: main fuel flow *WF*, nozzle critical area *AJ*, the compressor (fan) inlet variable guide-vane angle *CIVV*, and the rear compressor variable guide-vane angle *RCVV*. Three output variables were considered: compressor outlet (total) pressure *PTC*, turbine outlet (total) pressure PTT, and turbine outlet (total) temperature TTT. We used relative values and $\Delta t = 0.1$ s. Thus $n = 2$, $m = 4$, and $l = 3$. Figures 1 and 2 present the comparison between the linear and nonlinear simulated responses. These responses correspond to one operating point at a maximum PLA of 68 deg, without an afterburner. The time histories of all control variables *WF*, *AJ*, *CIVV*, and *RCVV* are presented in Fig. 1. The time response of the state variable *NC* is presented in Fig. 2. The uncertainties of the linear model are considered only for single-stage linear transitions. The matrices $\Delta A^{\max}$ and $\Delta C^{\max}$ only take into account the differences (uncertainties) between nonlinear and linear models and not uncertainties in the nonlinear model itself.

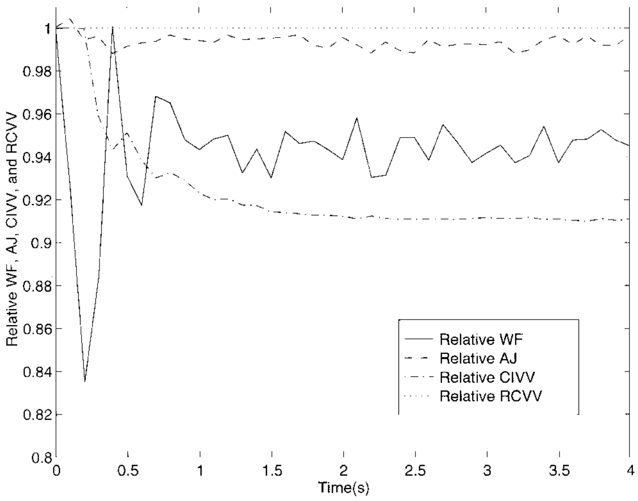

Fig. 1 *WF*, *AJ*, *CIVV*, and *RCVV* time histories.

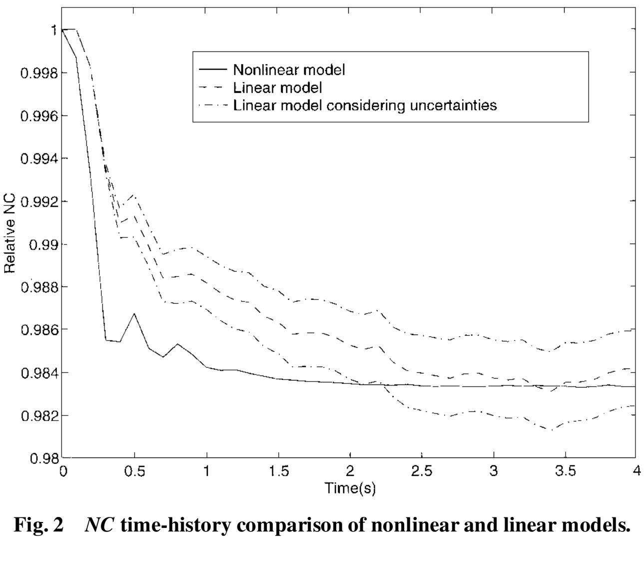

Fig. 2 *NC* time-history comparison of nonlinear and linear models.

Conclusions

A new estimation of the linear model matrix parameters method, along with matrix parameters limiting modeling errors (uncertainties or differences between nonlinear and linear models), was developed. This method enables us to obtain a set of linear models forming a piecewise linear model from a turbofan-engine-control-system detailed nonlinear model response in the time domain. The method uses linear programming and can consequently consider any additional linear constraints for the estimated matrix elements.

The results of this work may be applied to advanced turbofan engine and aircraft control systems as well as to other dynamic systems that can be described by linear or piecewise linear models by considering modeling uncertainty. Such models may be useful for robust optimal control design and for the development of sensor/actuator detection/isolation/accommodation algorithms.

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